JOURNAL OF AIRCRAFT
Vol. 40, No. 1, January–February 2003

Rational Objective Functions for Vehicles

David M. Paulus Jr.* and Richard A. Gaggioli†
Marquette University, Milwaukee, Wisconsin 53233

Although energy is the key resource for achieving vehicle performance desiderata, it is not necessary, nor is it necessarily desirable, to assess the desiderata in terms of energy alone. Yet, because the energy conversion systems in vehicles may use or supply energy in several forms (thrust, power, heat, cooling, etc.), exergy must be used to account for energy-based interactions between systems and for the costing of various "streams" (mass, power, etc.). Exergy is the key to the decomposition of energy systems and allows concurrent engineering of the several devices that may make up an overall system. One proposed method for creating an overall objective function is presented. Then, the methods of thermoeconomics will be combined with it to allow decomposition and detailed design of subsystems and devices. As an example, objective functions and weighting factors will be developed for a light experimental aircraft. Decomposition will be illustrated with the selection of an alternator and engine.

Nomenclature

A = area
AR = aspect ratio
C = lift or drag coefficient
c = unit cost
D = desideratum
F = feed
$\mathbf{F}$ = force
g = acceleration of gravity
J = total cost
m = mass
P = product; power
R = range
T = time weighting factor
V = velocity
v = unit value
W = weighting factor
$\dot{X}$ = exergy flow
Z = capital
Π = profit
ρ = density

Subscripts

D = drag
f = feed
L = lift
m = mass
p = product
T = thrust

Introduction

CURRENT practice in vehicular design is to list all of the desired objectives, for example, speed, payload, climb rate, acceleration, etc. and then to express some of them in quantitative terms and some solely in qualitative terms. However, there is not an objective procedure for weighting the relative importance of these often competing desiderata. The "weighting" is carried out, subjectively, during the course of the design process, primarily as a consequence of conversations between different involved personnel, such as the customer, the end-users, the chief engineer, the various specialized design teams, etc.

Although such conversations will always be important, to expedite the design process and make it more efficient, it would be desirable to establish a rational objective function for each overall vehicle project, including a quantitative means for assessing the importance of the various desired objectives.

Extensive work has been done in the area of optimized design of energy systems that have an end purpose of providing a mass, heat, or power flow. Examples include Refs. 1 and 2. However, many energy systems are used in applications for which the end product is not energy, such as vehicular applications. In such instances the energy systems impact not only lifetime costs of the application, but also its performance. It is necessary to take these performance impacts into account to optimize these energy systems properly. Indeed, for some vehicles, such as a fighter or a Formula-1 automobile, performance is the overwhelming objective. Often these performance desiderata are competing. For example, a modern fighter must excel in several performance areas. One objective is to have a low signature, that is, be stealthy. Another objective is rapid acceleration. The aircraft should have a long range and a great payload, as well as low cost and adequate production. The determination of the optimum balance between these desiderata is a greater challenge than many traditional energy system design problems.

The concepts developed here are applicable to vehicles in general, but will be specifically developed here for aircraft.

Although the authors believe that overall objective functions (as developed here), exergy analysis and thermoeconomics, could aid in preliminary design, they also believe that the aerospace industry as a whole does an excellent job making tradeoffs ("optimizing") to achieve an outstanding overall design. What is lacking is a means for deducing, from the overall objective function, subsidiary objective functions for detailed design (of devices, subsystems, etc.).

We advocate defining an overall objective function during preliminary design, even if it is not used for preliminary design. This allows standards to be set for the detailed design and/or selection of onboard energy conversion subsystems or devices. The development of this overall function specifically 1) generates weighting factors and 2) places values on performance desiderata. These two items allow thermoeconomic theory to be employed for decomposition and local optimization.

Proposed Overall Objective Function

Before beginning any design, an engineer (or team) must determine the constraints and objectives of the project. For a meaningful

Received 1 April 2001; revision received 8 July 2002; accepted for publication 17 August 2002.

*Ph.D. Student, Department of Mechanical and Industrial Engineering; currently Research Associate, Technische Universität Berlin, Instiut für Energietechnik, Marchstraße 18, D10587 Berlin, Germany. Member AIAA.
†Professor, Department of Mechanical and Industrial Engineering, 1515 West Wisconsin Avenue.

optimization, the objectives need to be expressed in a single function.

A traditional objective function for an overall energy system with a single feed and single product is[3]

$$J = c_f F + \sum Z - v_p P \tag{1}$$

where J is to be minimized. ($-J$ is the profit.)

Although a vehicle has energy feeds, its product is performance rather than energy. Nevertheless, Eq. (1) remains appropriate for the overall vehicle, when

$$J = c_f F + \sum Z - \sum v_p P \tag{2a}$$

Each P is now used to signify the various performance benefits. For the case of an aircraft, P might account for climb rate and cruising speed, for example.

Taking Eq. (2a) and negating it yields a "profit" to be maximized, which will be referred to as Π:

$$\Pi = \sum v_p P - \left[c_f F + \sum Z\right] \tag{2b}$$

This objective function is useful in conceptual and preliminary design, but can not directly be applied to systems or subsystems.

Determining the Value of Performance

Clearly, before one can apply Eq. (2b) to a conceptual design, one must find the numerical values of the various v_p. These values are, in the most general case, functions of the levels of performance themselves. One would expect a very low level of performance for a given desideratum would be zero, that is, it is unacceptable.

As an example of this, consider an automobile. A person residing in the United States (with typical speed limits in the 100–125 km/h range) might be unwilling to consider purchasing a vehicle incapable of at least 125 km/h.

However, this person would likely be willing to pay more for an automobile that could achieve a higher speed than this minimal amount. Speed limits (or, more correctly, the enforcement thereof) cause a reduction in the marginal value of a 1-km/h increase in speed as the top speed of the car becomes higher and higher. This hypothetical person, therefore, might not be willing to pay any more money for an automobile capable of reaching 240 km/h than an automobile capable of reaching 200 km/h. (Acceleration would be a separate desideratum.)

Returning to the example of the fighter plane, one can imagine that a certain level of overall performance could be reached that would insure victory over any opponent and penetration of any airspace. It is not rational to invest further resources in performance beyond this point. Thus, we expect that the functions $v_p(P)$ become flat after a certain value for a desideratum is reached. In between these points (the minimum acceptable and the maximum useful) lies some continuous function.

A method is laid out hereafter to estimate these functions. It consists of five steps and may involve iteration. Note that, ultimately, to optimize a subsystem, it is absolutely necessary to develop this information, that is, the functions $v_p(P)$, in some form, even if it is not done with the method used in this paper.

Step 1: Determining Median Performance

The foregoing algebraic "tradeoff functions" $v_p(P)$ for representing the value of performance may have an arbitrary shape between the points of minimum acceptable and maximum desirable performance. However, if a limited range of performance is considered, the assumption of a linear relationship is reasonable. One way in which this linear function may be constructed is around a median point. If a linear function is undesirable, this information will still be of use in the construction of the function.

For a military combat aircraft, one way to find a median point is by considering the performance of the aircraft's adversaries, both current and projected. (Here, the aircraft's adversaries are considered to be other aircraft. This view could, of course, be expanded to consider enemy air defense capabilities.) For each desideratum, at least one of the aircraft in the adversarial group has a best value. That value could be selected as the median of acceptable performance levels. The set of medians would form a "standard" of comparison for further investigations.

The same basic idea is applicable to civilian vehicles. However, the median values of a market segment might instead be chosen to set the standard values.

Step 2: Projected Units Costs and Projected Production

A realistic estimate of both the cost per vehicle and the total production quantity of the vehicle should be made. As will be seen, this step is an aid to the fourth step.

Step 3: Projected Research, Design, and Development Costs

The projected research, design, and development costs should be listed. This is also an aid to the following step. When employed with the information attained from steps 1 and 2, the total project cost may be estimated.

Step 4: Algebraic Tradeoff Functions

The algebraic tradeoff functions for each desideratum must be determined through questioning of the customer or end-user. As a bare minimum, the following three questions should be asked (in some form).

1) What is the minimum acceptable value for each of the desiderata?

2) Is there a point beyond which further improvement is not necessary?

3) How much would a given improvement, over and above the median value determined in step 1, be valued?

With the answers to these three questions determined, the simplest tradeoff function, linear, may be determined. Once again visiting the automobile example, let us imagine that the marketing department has asked these three questions to potential customers regarding the top speed of an automobile of a certain class. The average (or weighted average) answers were $S = 125$ km/h as a minimum acceptable top speed, $S = 200$ km/h as a ceiling beyond which improvement has little or no value, and a willingness to pay \$800 for an improvement $\Delta S = 10$ km/h over a median 165 km/h top speed. One can imagine a function $v_s(S)$ that would have a value of \$0 up to 125 km/h and rise with a slope of \$80/(km/h) to a maximum of \$6000.

However, for many cases the questions may not be best asked in terms of dollars. For a military vehicle, say an air superiority fighter, dollars would be a poor choice of units. In this example, the end-user (an air force or navy) is not the same as the purchaser (Congress). The purchase costs have reached such high amounts that it is difficult for the average person to comprehend the sums in rational terms. Furthermore, neither body (Congress or the end-user) is spending their own money.

In such a case the questions may be rephrased in terms of production sacrifices; question 3 could be changed to the following: "What reduction, in number of aircraft delivered to you, would you accept to obtain a specified improvement from the median value determined in step 1?" A military leader should have a good grasp of tradeoffs between quantity and quality. The information from steps 2 and 3 allows production tradeoffs to be converted to a dollar amount (or a cost to a production adjustment).

If a computer simulation were to be available that would predict aircraft survivability as a function of measured desiderata, it could be used to develop, or help develop, these tradeoff functions.

Step 5: Relative Weighting

At the overall vehicle level, the algebraic tradeoff functions are sufficient for optimization. As shown hereafter, to decompose the vehicle into systems and subsystems, weighting factors will be needed. Then a distinct objective function can be defined for each system, subsystem, or device, each to be designed by a distinct "team."

The relative weighting factor for the desideratum D_i, as employed in this paper, is deduced from the foregoing information and is given by

$$W_i = \frac{(\Delta X_i/Z)/(\Delta D_i/D_i)}{\sum_k(\Delta Z_k/Z)/(\Delta D_k/D_k)} \tag{3}$$

where ΔZ the increase in price an end-user would pay for a ΔD increase in performance. The numerator, then, represents the percent of expenditure increase the customer is willing to make, per percent increase in performance desideratum D_i. The denominator is the sum over all performance desiderata, so that W_i represents the relative importance of D_i. Z cancels, leaving

$$W_i = \frac{\Delta Z_i/(\Delta D_i/D_i)}{\sum_k \Delta Z_k/(\Delta D_k/D_k)} \tag{4}$$

Iteration

It may be necessary to repeat steps 1–5 as a preliminary design is completed. This is due to an implicit assumption as to the independence of the individual desiderata. An end-user may sacrifice far more total resources than intended because this person was looking at only a single desideratum at a time when proceeding through the five steps. On the other hand, the standard performance may be so great that the user erred in the opposite direction. Therefore, there must be good communication between designer and end-user at all times. As a conceptual design evolves, the questions from step 3 may need to be repeated. A design may have become, in the end-users opinion, too expensive, or its performance may simply be inadequate. This is especially true if the median performance was far from the optimum.

Notes on the Development of an Overall Objective Function

In the methods presented here, the units of each term in the objective functions are monetary. For commercial vehicles and transport vehicles, monetary units are the obvious choice. For combat vehicles, other units may be better, such as "production quantity."

Concurrent Engineering and the Need for Decomposition

Once a conceptual design has been accepted, the detailed design should proceed in an efficient fashion. It is not possible to optimize and design an aircraft, or even an energy subsystem, as an entirety. Vehicular energy systems are too complex to be designed by a single individual or team because the number of decision variables becomes unmanageable. Therefore, the design or selection of individual devices and/or subsystems is delegated to subordinate teams or individuals. It is desirable for these designers to have a methodology and information that allows them to make decisions in accordance with the overall goals of the vehicle. Furthermore, it must be determined which decision variables are local and which are global; that is, which variables a device design team is free to vary in optimizing its individual device, and which are constraints to the design team so as not to affect the designs being carried out, simultaneously, by other design teams. These determinations may at times be made through common sense, but at other times may require a sensitivity analysis.

A vehicle typically relies on one fuel to achieve its goals. Because the subsystem or device design team is not optimizing a whole vehicle, but something that may produce or consume commodities not considered when looking at the whole vehicle, its objective function will vary from that of the overall vehicle. For example, the alternator on a light general aviation aircraft does not directly consume fuel, nor does it directly influence performance. Nonetheless, an aircraft with an alternator that is both lighter and more efficient, will, with all else remaining equal, perform better (and cost more). The person selecting an alternator should not be burdened with, nor at this stage of the design be necessarily capable of, directly calculating the impact on aircraft performance.

The aforementioned alternator still has only one product (electrical energy). An onboard energy device or subsystem may use several "fuels" and/or supply several "products." These fuels and products, besides electricity and shaft power, include, but are not limited to, compressed air, hydraulic power, and heat (or cooling). Additionally, one must account for the lift required to hold the device in the air (or to make it go up). The costs and benefits of these mass and energy flows must be included in the device or subsystem design team's objective function.

Before proceeding in developing an objective function, for a device or subsystem, which is to be designed by a team subordinate to the overall vehicle design team, methods are needed for determining the costs and benefits. This is done through accounting for mass and energy streams with exergy and using the methods of thermoeconomics to place unit costs on them. (As a general reference to exergy and thermoeconomics see, for example, Ref. 4.)

Role for Exergy and Thermoeconomics

The role for exergy and thermoeconomics discussed within this paper is for employment during detailed design. In this paper, detailed design is the filling in of black boxes such as a propulsion system, environmental control system or a hydraulic pump. The methodology presented here assumes that the overall vehicle designers have penned a reasonably optimized preliminary design. Additionally, it is assumed that allowance has been made for the weight and exergy consumption of systems and subsystems. Before employing this thermoeconomic methodology, it is absolutely necessary to have the overall objective function, weighting factors, and values of performance defined earlier. Thus, all design teams will strive for the same common goals.

Exergy

For comparison of energy converters that have single feed and product streams, and that perform the same function, performance measures such as "thermal efficiency" and "coefficient of performance (COP)" suffice. Such energy-based performance measurements are referred to as first-law performance measures. As devices or systems add additional feeds and/or products, these definitions become inadequate. Moreover, even if a device has a single feed and a single product, but is one component in a complex system, such first-law measures are inadequate for comparing the relative impact of any one device on the overall system performance. That is, first-law measures of feeds and products do not represent the true values of these commodities. What is needed in analyzing and optimizing complex systems is a common, consistent measure for representing the potential of mass and energy flows.

This proper method of quantifying fuels and products is with exergy (sometimes called "availability" or "available energy"). Exergy measures the ability of a commodity (matter, momentum, entropy or heat, charge, etc.) to cause change.

After preliminary design, all exergy flows and destructions within the proposed vehicle should be diagramed. Because vehicle may be evaluated under several different operating modes (typically related to the desiderata), it may be necessary to create several exergy flow diagrams. If one were developing a fighter aircraft, diagrams may be necessary for cruise, maneuver, climb, etc.

Exergy is dependent on the environment in which the vehicle operates. If the range of operating conditions is not too large, an average (or weighted average) of the operating conditions may suffice to serve as the reference environment. For vehicles operating in a wide variety of conditions, that is, aircraft, a number of diagrams, sufficient to cover the variations in operating environment, will be necessary. It may even be necessary to express values as functions of time for such realms of operation as a maximum performance climb, where ambient pressure and temperature are changing rapidly.

The purpose of these exergy diagrams is twofold. (See, for example, Refs. 5 and 6.) The first benefit is that this diagram allows the designers truly to comprehend the interactions and inefficiencies of an energy system. The relative magnitude of the destructions (and wastes) will point to the areas of a system where improvements will be most beneficial. This topic will not be addressed directly here, but is addressed extensively in the literature. The second benefit of creating the exergy flow and destruction diagram is that costs may be assigned to each stream's exergy flow. That is, the cost per unit of exergy can be evaluated at each station on the exergy flow diagram. Thus, not only exergy, but also costs, can be tracked through the overall vehicle. Moreover, the unit costs of exergy entering a subsystem (or device) are of special relevance to that subsystem's design team. The methods for establishing the units costs, thermoeconomics, will now be discussed.

Thermoeconomics

Thermoeconomics allows a cost to be associated with each exergy flow. These unit costs are absolutely necessary in the development of the objective functions for an onboard device or subsystem.

After the overall exergy flow diagrams are complete, complete exergy costing and cost flow diagrams should be created for each exergy flow diagram. Capital costs should not be included in the costing; because the local optimization should be changing design values only by relatively small amounts, the proper monetary charges are for marginal costs.

For a subsystem, the costing will be performed one of two ways, depending on whether it is at a maximum performance condition or a cruise condition. These correspond to costing for fixed resources or fixed output, respectively.[3] In a maximum performance condition, the vehicle's primary energy conversion system, that is, its propulsion system, is producing as much power as possible. Any exergy drawn to fuel the subsystem will be detracting from the vehicle's performance. Therefore, all exergy flows must be assigned a per unit cost based on the cost to vehicle performance. For example, a light aircraft in full throttle cruise at a specified altitude, where the desired performance is a cruise speed, can have a marginal value of thrust exergy assigned with

$$v_T = \frac{\partial V}{\partial \dot{X}_T}\frac{\partial \Pi}{\partial V} \tag{5}$$

Here, $\partial V/\partial \dot{X}_T$ is the partial derivative of velocity with respect to thrust exergy and $\partial \Pi/\partial V$ is the slope of the cruise speed tradeoff curve. Multiplying these two partial derivatives yields $\partial \Pi/\partial \dot{X}_T$, which clearly is the marginal profit associated with thrust exergy. This information allows the complete exergy costing to be completed.

On the other hand, for the case of "part-throttle" cruise, costing is based on the cost of fuel. Engine power is no longer the limiting commodity; any drop in performance may be accommodated by increasing the power output of the engine. One resulting cost is the direct cost for additional fuel. Furthermore, if range is a desideratum, the cost of an increase of fuel burn to range may be added to the fuel's base cost. This will, in general, be

$$c_{f,\text{range}} = -\frac{\partial R}{\partial \dot{m}_{\text{fuel}}}\frac{\partial \Pi}{\partial R} \tag{6}$$

If a device is a member of the vehicle's propulsion system, for example, a piston engine in a propeller driven aircraft, it will be shown that exergy costing will have to be done from both of the preceding viewpoints.

The application of the second law of thermodynamics, combined with economics, allows for decomposition of systems into devices and subsystems that may be independently optimized. This theory has been developed and validated by, for example, El-Sayed and Evans[1] and El-Sayed.[3]

Objective Function for an Onboard Subsystem

With the exergy flow and cost diagrams complete, there is enough information to optimize a subsystem. Here a subsystem is defined as an onboard energy conversion system not directly involved in producing the necessary basic forces of flight: thrust and lift. The subsystem is, in general, required to deliver a certain output as a design constraint. Then, the local objective function for a subsystem, to be minimized, is a total cost.

$$J = Z + \sum_i W_i \left(c_{mi} m + \sum_{i,j} c_{i,j} \dot{X}_{i,j} \right) \tag{7}$$

Here, i represents each desideratum, j represents each feed stream, $c_{i,j}$ are unit costs of exergy, c_m is the unit cost of mass, and m is the mass of the component. For an aircraft, the cost of mass is related to the exergy of lift. As was stated earlier, this exergy expenditure used to hold mass in the air must be taken into account. Suitable relations will be given in the example that follows. W represents the various weighting factors. In general, costs will be functions of the decision variables themselves. For example, the marginal cost of weight onboard an aircraft is a function of the aircraft's gross weight. Therefore, as a subsystem designer strays from the estimates made about the subsystem during preliminary design, iterations may be necessary to update exergy and weight costs.

Objective Function for a Propulsion System Component

Whereas a subsystem, in general, will have as its required output a design constraint, the propulsion system itself is a balance between performance supplied to the aircraft (as thrust), weight, capital costs, range, and operating costs. Its objective function differs from that of a subsystem in that there is now a profit to be maximized, the difference between the performance delivered and the cost of providing it. This takes the form of a profit to be maximized:

$$\Pi = \sum_i W_i \left(\sum_{i,j} v_{i,j} \dot{X}_{i,j} - c_{mi} m \right) - Z - \sum_i T_i \sum_{i,k} c_{i,k} \dot{X}_{i,k} \tag{8}$$

It is necessary to cost exergy in both manners discussed earlier. Exergy outputs, with subscript j, are determined by placing a value on thrust. Inputs, with subscript k, are determined using the price of fuel (including, if necessary, cost adjustment for range). The variable T is a time factor, the percentage of time a vehicle spends in each operating mode. This follows from the overall vehicle's objective function; we charge a device in the propulsion system for only the fuel consumption for which it is responsible. In contrast, performance is available all of the time; therefore, the output values are multiplied by the weighting factors of the various desideratum.

Application to Alternator and Engine Selection

The preceding principles will be applied to a real example for an actual aircraft. The aircraft here is the Glastar, an aircraft built from a kit manufactured by New Glastar (previously Stoddard–Hamilton). It is a light, high-wing aircraft, with a gross takeoff weight of 889 kg (1960 lb), 93–149 kW (125–200 hp), a cruise speed of about 260 km/h (140 kn) and a climb rate of 457 m/min (1500 ft/min) at gross weight on 119 kW (160 hp). Typical missions for this aircraft include short field work, cross-country, and pleasure flying. It may be equipped with conventional or tricycle gear, or floats or skis.

With such a kit, the builder is responsible for designing the aircraft forward of the firewall. (The following is one step in the construction manual: Install everything forward of the firewall.) Furthermore, the builder is also, in general, the end-user.

The problems addressed here will be the selection of an alternator (a subsystem) and the engine (a device in the propulsion system). Because the purchaser has a large choice of kits from which to choose, it is logically assumed that the chosen aircraft is near to an optimal solution. This aircraft kit, with a 160-hp piston engine and a constant-speed propeller, is considered the preliminary design. These two decisions are then steps in the detailed design process. Although this aircraft is simple, these examples are solutions to real design problems.

Additionally, these examples are applied to discrete choices. In the majority of instances in which detailed design of energy systems is occurring, the engineer is relegated to discrete choices because items are not available in an infinite number of sizes. This should not be construed to mean that the objective functions presented here are valid only for discrete decision variables.

User Responses and Operating Modes

To construct objective functions it was necessary to question the user about the value of different desideratum.

It was decided, based on the aircraft's typical mission profile, to consider full-power cruise at 8000 ft, economy cruise at 8000 ft, and climb rate as desiderata. Climb rate was considered a measure of short field performance. Although it would have been advantageous to add a more direct measure of this part of the mission, it would have involved using a far more complicated aircraft simulation. A summary of conditions for the measurement of the various desiderata and their preliminary design values is given in Table 1. Note that the values in Table 1 are from a simple aircraft performance model. The values computed for speed and range are very close to manufacturer's test data. However, climb rates calculated by the program appear approximately 300 ft/min too low. For the purpose of the example, values from the model will be used as is.

The following questions were posed to the end-user, and the following answers received.

1) When you were selecting an aircraft kit, what was the range of advertised cruise speeds of the aircraft (at 75% power, 8000 ft) in which you were interested? The answer was 120–200 mph.

2) Approximately how much money would you be willing to spend to increase the cruise speed of your airplane by 5 mph above the average of the values in question 1? The answer was $350.

3) When you were selecting an aircraft kit, what was the range of advertised ranges of the aircraft in which you were interested (at 65% power, 8000 ft)? The answer was 500–1000 miles.

4) Approximately how much money would you be willing to spend to increase the range of your airplane by 50 miles over the average of the values in question 3? The answer was $300.

5) When you were selecting an aircraft kit, what was the range of advertised climb rates of the aircraft in which you were interested? The answer was 700–1800 ft/min.

6) Approximately how much money would you be willing to spend to increase the climb rate of your airplane by 100 ft/min above the average of the values in question 1? The answer was $250.

With this information, linear functions for the three desiderata were created. They are shown in Figs. 1–3.

Table 1 Preliminary design values

Desiderata	Cargo mass, kg	Fuel mass[a], kg	Speed, km/h	Climb, m/min	Fuel burn, kg/h
Full power cruise	204	54	259	0	22.9
Economy cruise	221	54	244	0	18.8
Climb	221	109	147	372	38.1

[a]Although exergy and performance calculations for economy cruise were performed with half-fuel to approximate an average, range calculations were performed with the flight beginning with full fuel (109 kg).

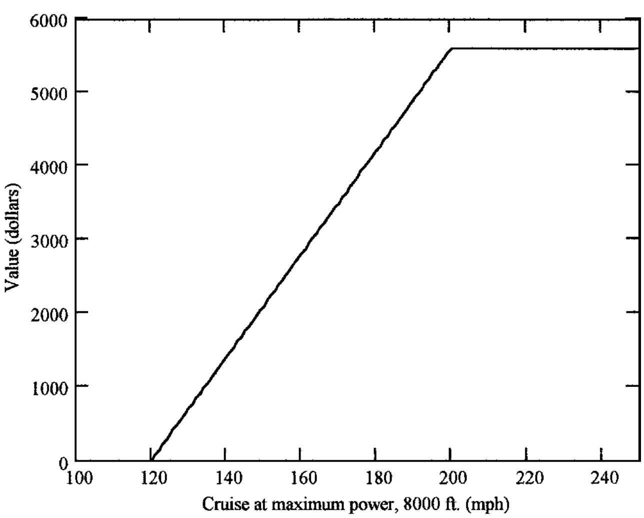

Fig. 1 Value of speed.

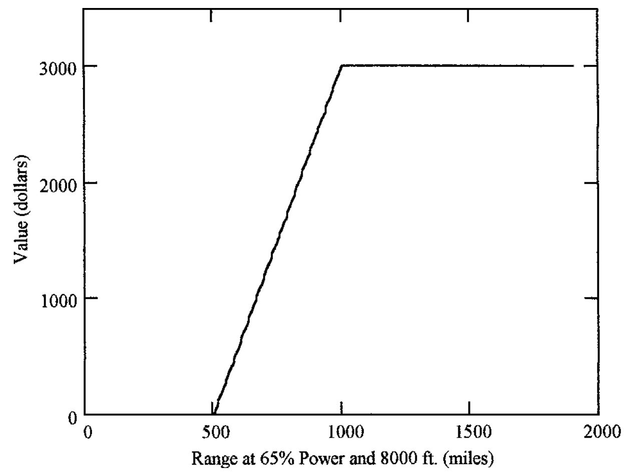

Fig. 2 Value of range.

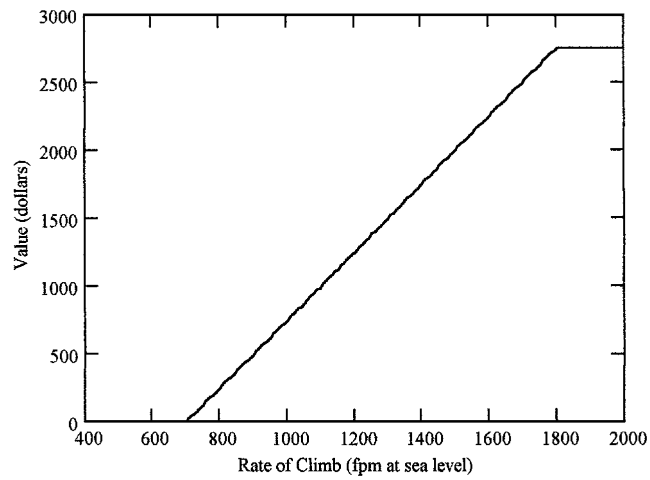

Fig. 3 Value of climb rate.

For all subsequent calculations, it was assumed that the aircraft would have a lifespan of 20 years, be flown 200 h/year, with 50% of those hours at economy cruise and the remaining 50% at high-speed cruise. Hours spent in climb are considered negligible. An interest rate of 8% was used.

Exergy Flow in a Light Aircraft

The Glastar aircraft will be broken down into the following components for the purpose of the exergy flow diagrams: 1) onboard fuel, 2) engine, 3) propeller, 4) alternator, 5) wing, and 6) fuselage and miscellaneous (including stabilizer and trim drag destructions).

To create the exergy flow and costs diagrams that have been referred to, certain new exergy relationships must be developed.

Exergy of Lift

Although the exergy produced by the wing in level cruise (at the expense of drag, that is, through an input of exergy with momentum) is ultimately destroyed, it is necessary to calculate the value of the lift exergy to charge individual components properly for their lift requirements.

In general, for any force, the exergy transfer associated with it may be defined as

$$\dot{X}_{\text{force}} = \boldsymbol{F}(\boldsymbol{V} - \boldsymbol{V}_0) \tag{9}$$

During steady flight, the force of lift is equal to the weight of the airplane (or the component being lifted). The problem remaining is to define $\boldsymbol{V}_0$ properly. An ideal wing, producing no entropy, traveling forward at velocity V_x will fall toward the Earth at a certain velocity V_{y0}. This is a thermostatic state, as defined by Gaggioli et al.[7]

McCormick[8] states that the minimum induced drag coefficient for a wing is

$$C_{Di,\min} = C_L^2 / \pi a \tag{10}$$

Therefore, the minimum thrust required to generate lift is given by

$$F_D = \frac{\rho A V^2 C_L^2}{2\pi a} \tag{11}$$

The power to maintain level flight with this ideal wing is then, in accordance with Eq. (11),

$$\frac{\rho A V_x^3 C_L^2}{2\pi a} = P_{\text{req,ideal}} \tag{12}$$

The rate of climb for a typical light aircraft is well approximated with

$$V_y = (P - P_{\text{req}})/mg \tag{13}$$

The use of Eqs. (9), (12), and (13) yields the following expression for the exergy of lift:

$$\dot{X}_{\text{lift}} = mgV_y + \frac{2(mg)^2}{\rho V_x A \pi a} \tag{14}$$

For an aircraft in steady flight, the total lift must equal the total weight of the aircraft. Therefore, it is logical to postulate that the lift generated by the wing will be consumed by individual components of an aircraft proportionally to their weight. Then, for component, i, it can be said that

$$\dot{X}_{\text{lift},i} = (m_i/m_{\text{total}})\dot{X}_{\text{lift}} \tag{15}$$

This individual exergy influx to a component due to lift requirements must be accounted for in a component's exergy balance.

From Eq. (14), the marginal cost of mass is

$$c_m = \frac{\partial \dot{X}_{\text{lift}}}{\partial m} c_{\text{lift}} \tag{16}$$

Note that c_{lift} is the unit exergy cost of lift, not the coefficient of lift. Performing the differentiation yields

$$c_m = gV_y + (4g^2 m/\rho V A \pi a)c_{\text{lift}} \tag{17}$$

Exergy Flow Diagrams

Figures 4, 5, and 6 show the exergy flows and per unit exergy costs in a light aircraft for full power cruise, economy cruise, and climb. Exergy destructions are also given.

Figures 4–6 were developed by 1) writing an exergy balance on each device and 2) writing cost balances on each device and adding auxiliary equations. (In the interest of conserving space, the systems of equations are not listed here. A good explanation of the methods employed to develop the equations, along with simple examples, is found in Ref. 9, pages 3–50. An additional valuable source is Ref. 4.) Additionally, it is necessary to fix a unit cost of exergy; this was done with either Eq. (5) or with fuel cost and Eq. (6), depending on the flight conditions.

Additionally, engine and propeller performance relations were used. These were in the form of a user's manual for the Lycoming engine and a computerized propeller map supplied by Hartzell. Aircraft performance was estimated from airfoil and manufacturer's test data.

Decomposition: Selection of an Alternator

Here it will be decided whether it is better to purchase a standard alternator or a lightweight model. Because no efficiency data are available, both will be assumed equally efficient. The standard alternator has an initial cost of $294 and a mass of 5.9 kg (13 lbm). The alternative costs $450, but has a mass of only 2.7 kg (6 lbm).

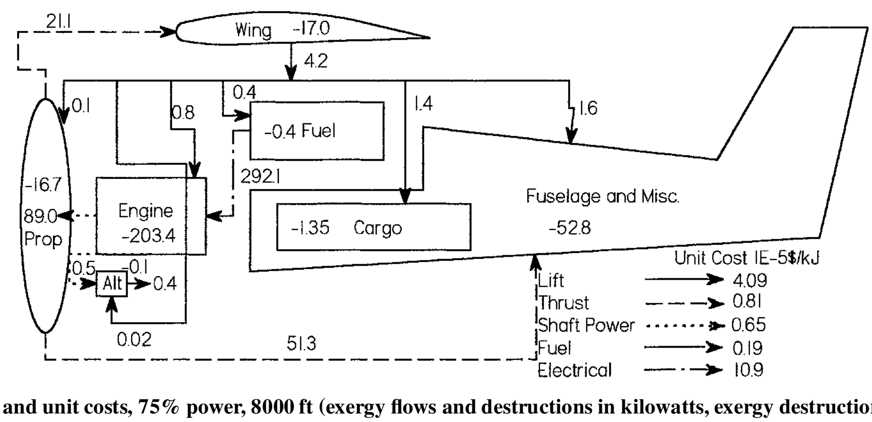

Fig. 4 Exergy flows and unit costs, 75% power, 8000 ft (exergy flows and destructions in kilowatts, exergy destructions are negative values).

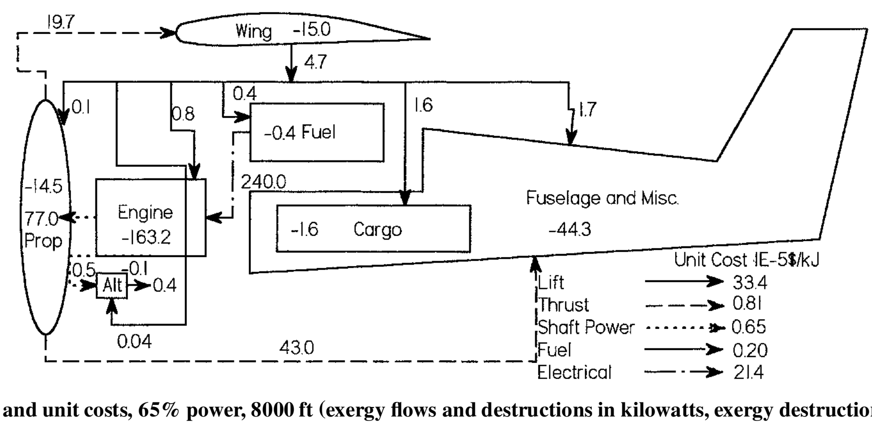

Fig. 5 Exergy flows and unit costs, 65% power, 8000 ft (exergy flows and destructions in kilowatts, exergy destructions are negative values).

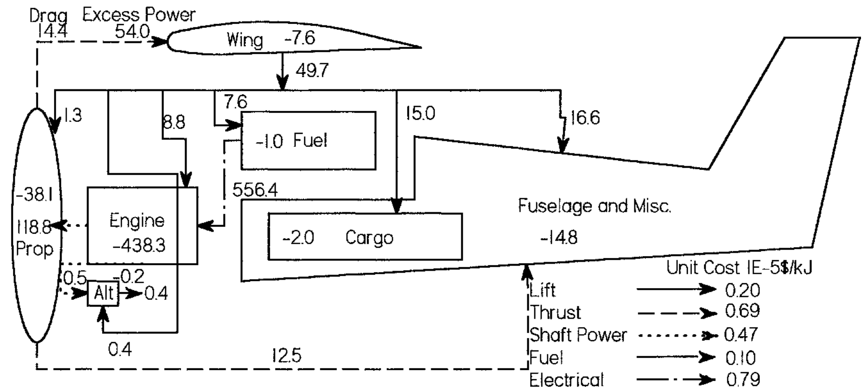

Fig. 6 Exergy flows and unit costs, full power, sea level, maximum rate of climb (exergy flows and destructions in kilowatts, exergy destructions are negative values).

Table 2 Performance with different engines

Engine, hp	Cost, thousands of dollars	Mass, kg	Full power cruise, km/h/fuel burn, kg/h	Fuel burn at 244 km/h, kg/h	Rate of climb, m/min
160	22.3	116	259/22.9	18.8	372
180	26.2	122	271/29.9	21.8	442

To make this decision, Eq. (7) is used. Terms for shaft power are unnecessary because the power inputs to both units are the same. The cost of switching from the standard to the lightweight alternator will be given by

$$\Delta J = Z_{\text{lightweight}} - Z_{\text{standard}} + \sum_i W_i c_{mi} (m_{\text{lightweight}} - m_{\text{standard}}) \tag{18}$$

The lifespan of 20 years, with an interest rate of 8% and 200 operating hours per year yields \$0.229 and \$0.150/h for values of Z for the lightweight and standard alternators, respectively.

The performance values from Table 1, along with the information from Figs. 1–3 (with slopes of \$350/5 mph, \$300/50 miles and \$250/100 ft/min), yield weighting factors of 0.565 for high-speed cruise, 0.283 for range and 0.153 for climb rate.

Equation (17) is used to find the costs of mass, with the cost of lift taken from the preceding exergy flow diagrams. This yields marginal mass costs of \$4.17 × 10^{-4}, \$3.33 × 10^{-3}, and \$5.32 × 10^{-4}/kg · h for full power cruise, economy cruise, and rate of climb, respectively.

Finally, evaluating Eq. (18) with these values yields an increase in cost for the aircraft of \$0.0608/h or a present value of \$118, associated with the lightweight alternator. Therefore, it is not desirable to use the lightweight alternator over the standard model.

Evaluating the aircraft performance by simulating the aircraft with the model used for design point calculations, along with the overall objective function in Eq. (2) and the functions shown in Figs. 1–3 yields an increase of cost of \$121 for the lightweight alternator. In terms of overall aircraft cost, or even alternator cost, the difference between the values found through decomposition and through overall simulation is insignificant.

Decomposition: 180 Versus 160-Horsepower Engines

The 160-hp engine is the standard engine, chosen most widely by builders. The 180-hp engine will produce gains in full-power cruise and climb rate, but at the expense of increased weight, decreased range, increased fuel consumption and increased capital cost (Table 2).

This information may be applied directly to the overall objective function, which results in decrease of profit (present value) of \$10,800.

Alternatively, Eq. (8) may be employed in the form

$$\Delta J = \sum_i W_i \left[\sum_{i,j} v_{\text{power},i} (\dot{X}_{\text{power},i,180} - \dot{X}_{\text{power},i,160}) - c_{mi}(m_{180} - m_{160}) \right] - (Z_{180} - Z_{160}) - \sum_i T_i \sum_i c_{\text{fuel}_i} \dot{m}_{\text{fuel}_i} \tag{19}$$

The unit costs of power, that is, exergy may be taken from the exergy flow diagrams, and the weighting factors remain the same as in the preceding examples. The time factors are 0.5 for full power cruise, 0.5 for economy cruise, and 0 for climb. Fuel costs are calculated on a per unit mass basis here because fuel is the raw energy source. The value used is \$0.735/kg. (Exergy costing with the fuel exergy costs assigned would yield the same values.) The cost of fuel burn to range must be added to the cost of fuel for evaluation of range at economy cruise, per Eq. (6), increasing the per kilogram cost of fuel to \$0.912.

Evaluation of Eq. (20) shows a decrease of present value profit of \$11,400 for the 180-hp engine. Although this differs from the \$10,800 nondecomposed value by 5.5%, the same decision would be reached. The error likely results from the large shift from the preliminary design values, shifts in engine power of 12%, (engine) weight of 6%, and fuel consumption of 14%. In such cases, iteration in the exergy costing step would improve results if necessary.

Conclusions

A logical method has been laid out for 1) generating objective functions for vehicles in general and aircraft in particular and 2) applying this objective function, along with exergy methods and thermoeconomics, to optimizing decomposed onboard energy conversion systems. These methods should be a useful tool to the detailed design stage. They allow a subsystem or device designer to maintain overall vehicle goals during detailed design, without recourse to continual vehicle simulation.

Without an overall objective function, it is impossible to decompose an aircraft into systems, subsystems, and devices to be designed in such a manner that independent optimization of each will also optimize the overall vehicle.

This decomposition method was applied to two examples for a simple aircraft, an energy conversion subsystem (an alternator), and a device in the propulsion system (a piston engine). The differences between optimal values calculated through decomposition and overall simulation were minimal for the alternator, and acceptable for the engine because the same decision would be made with either method. This is despite a large shift from the preliminary design.

Further work to be performed includes the testing of the proposed methods to progressively more complicated aircraft and systems. It is in this area, where simulation becomes far more time consuming and the decision variables multiply, that the decomposition strategy will show its true value. A refinement of the techniques presented here would include evaluating sensitivities of decisions to the end-user's answers to the questions used in evaluating values of performance.

References

[1]El-Sayed, Y. M., and Evans, R. B., "Thermoeconomics and the Design of Heat Systems," *Journal of Engineering for Power*, Vol. 92, ASME, New York, Jan. 1970, p. 27.

[2]Evans, R. B., and von Spakovsky, M. R., "Second Law Aspects of Thermal Design," Vol. HTD 33, ASME, New York, 1984, p. 27.

[3]El-Sayed, Y. M., "Repowering Second-Law-Based Optimization," AES-Vol. 35, ASME Press, New York, 1995, pp. 405–421.

[4]Bejan, A., Tsatsaronis, G., and Moran, M. J., *Thermal Design and Optimization*, Wiley Interscience, New York, 1996.

[5]Moran, M. J., *Availability Analysis: A Guide to Efficient Energy Use*, ASME Press, New York, 1989.

[6]Szargut, J., Morris, D. R., and Stewart, F. R., *Exergy Analysis of Thermal, Chemical and Metallurgical Processes*, Hemisphere, New York, 1988.

[7]Gaggioli, R. A., and Paulus, D. M., Jr., "Available Energy: II. Gibbs Extended," AES-Vol. 39, ASME, New York, 1999, pp. 285–296.

[8]McCormick, B. W., *Aerodynamics, Aeronautics and Flight Mechanics*, Wiley, New York, 1995.

[9]Gaggioli, R. A. (ed.), *Efficiency and Costing*, American Chemical Society, Washington, DC, 1983, pp. 3–50.